

How long-range interactions tune the damping in compact stars

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Long-range interactions lead to non-Fermi liquid effects in dense matter. We show that, in contrast to other material properties, their effect on the bulk viscosity of quark matter is significant since they shift its resonant maximum and can thereby change the viscosity by many orders of magnitude. This is of importance for the damping of oscillations of compact stars, like in particular unstable r-modes, and the quest to detect signatures of deconfined matter in astrophysical observations. We find that, in contrast to neutron stars with standard damping mechanisms, compact stars that contain ungapped quark matter are consistent with the observed data on low mass x-ray binaries.

Quantum chromodynamics (QCD) features strong, long-range gauge interactions that lead to color confinement. At high density these interactions are partly screened by the medium and can lead to deconfinement and various hypothetical forms of quark matter [1]. In the color-flavor locked (CFL) phase at asymptotically large density all quark modes are gapped, but many of the possible phases at moderate densities that might be reached in compact stars feature ungapped quark excitations. Yet, in-medium effects cannot entirely screen the gluonic interaction, but their magnetic part is only Landau-damped [2]. These long-range correlations strongly modify the low-energy fermionic excitations of deconfined dense matter and lead to non-Fermi liquid (NFL) behavior [3, 4] of various material properties [5–8]. Analogous NFL effects due to long-range interactions are studied extensively in condensed matter systems [9] and with dual holographic descriptions [10]. Whereas in such systems NFL effects are generally only found in controlled experiments at low temperatures, we discuss here a case where they have important observable consequences outside of a physical laboratory and in an environment hotter than the surface of the sun.

Compact stars are the only known objects that are dense enough that they could contain deconfined quark matter, and their mechanical oscillation modes are the only known way to directly probe the dense matter in their interior. The connection is established via the damping of these modes which is locally described by the viscosities of the particular microscopic form of matter. Whereas the shear viscosity is generally very similar in hadronic and quark matter, since both arise from unscreened plasmon-enhanced scattering of relativistic particles, namely leptons [11] and ungapped quarks [5], respectively, the bulk viscosity can be decisively different for various forms of matter [12–17]. In dense matter bulk viscosity arises generally from slow flavor-changing weak interactions. The bulk viscosity is a resonant mechanism that features a sharp maximum when the time scale of the relevant weak process matches the one of a driving density oscillation. Although the bulk viscosity is induced by weak processes, their rate depends on strong interactions since they can open phase space for weak

transitions. Correspondingly low energy strong interactions control the resonance, analogous to the dial of a radio tuner, and even moderate changes can alter the viscosity at a given temperature and frequency by many orders of magnitude.

In this work we study the influence of NFL interactions on the bulk viscosity of dense quark matter. The bulk viscosity stems from a lag between a driving density oscillation and the chemical equilibration of the system mediated by slow weak interactions. Here we study the damping of small amplitude mechanical oscillations in the subthermal case where the displacement of the chemical potentials μ_Δ from their equilibrium values satisfies $\mu_\Delta \ll T$. The suprathreshold regime [13, 16] and the influence of strong interactions on the saturation of unstable modes [18] will be discussed elsewhere. Whereas in dense hadronic matter chemical equilibration is caused by semi-leptonic Urca processes, in dense quark matter strangeness-changing non-leptonic processes $d+u \leftrightarrow u+s$ dominate [13]. In this case the displacement $\mu_\Delta = \mu_d - \mu_s$ is finite when density oscillations drive the system out of chemical equilibrium and causes a net non-leptonic rate $\Gamma_{nl}^{(\leftrightarrow)} \equiv \Gamma_{d \rightarrow s} - \Gamma_{s \rightarrow d}$ that re-equilibrates the system. To compute this rate we follow the analysis performed in [19] and extend it to the interacting case. The computation requires the dispersion relations of low energy quarks which is modified by self-energy corrections from quantum fluctuations involving unscreened magnetic gluons. The form of the dispersion relation including such NFL effects is close to the Fermi surface given by [3, 7, 20]

$$p_i \approx p_{Fi} + v_{Fi}^{-1} \left(\left(1 + \sigma \log \left(\frac{\Lambda}{E_i - \mu_i} \right) \right) (E_i - \mu_i) - \delta \mu_i \right) \quad (1)$$

where in the weak coupling, hard dense loop [21] approximation the parameters are given by [6, 7]

$$\begin{aligned} v_F &\approx 1 - \frac{2\alpha_s}{3\pi} = \quad, \quad \delta\mu \approx \frac{4\alpha_s}{3\pi} \mu \\ \sigma &\approx \frac{4\alpha_s}{9\pi} \quad, \quad m^2 \approx \frac{2\alpha_s}{3\pi} \mu^2 \quad, \quad \Lambda \approx 0.28m \end{aligned} \quad (2)$$

The strong correlations significantly increase the density of states near the Fermi surface where the group veloc-

ity of the corresponding excitations gradually deviates from its Fermi liquid value v_F and eventually vanishes in the low energy limit. This effect arises from a kinematic low energy enhancement and is even present in the weak coupling regime. Yet, the parametric form of the NFL dispersion relation has been shown to hold beyond perturbation theory [3] and via a manifest power counting analysis it has been pointed out that this result holds within an effective theory in the low temperature limit even at strong coupling [4]. Aside from the strong generic increase of the density of states close to the Fermi sea due long-range interactions, the other strong interaction corrections to the Fermi liquid parameters are not logarithmically enhanced and due to our ignorance on the values of these parameters in the strong coupling regime relevant for compact stars we neglect these moderate corrections here. As discussed in [7] there are likewise no logarithmic corrections to the weak interaction vertices.

In the subthermal limit $\mu_\Delta \ll T \ll \mu$ and neglecting the subleading quark masses $m_i \ll \mu$ we find for the non-leptonic rate

$$\Gamma_{nl}^{(\leftrightarrow)} \approx -\frac{64G_F^2 \sin^2 \theta_c \cos^2 \theta_c}{5\pi^3} \mu_q^5 T^2 \left(1 + \sigma \log\left(\frac{\Lambda}{T}\right)\right)^4 \mu_\Delta \quad (3)$$

This result differs from the non-interacting expression given in [13, 19] by the strong interaction corrections in terms of NFL factors $\lambda \equiv 1 + \sigma \log(\Lambda/T)$ for each of the involved four quarks. These factors become large in the ultradegenerate regime $T \ll \Lambda = O(\mu)$ realized in compact stars. In eq. (3) additional terms suppressed in powers of σ/λ which could not be evaluated in closed form have been neglected, but a numerical integration shows that their combined contribution is at the relevant temperatures less than 5%.

For comparison we will also study the viscosity due to Urca processes including NFL interactions. The direct Urca rate is obtained as a generalization of the Fermi liquid expression [17, 22]. In case of the dominant d -quark Urca processes $d \rightarrow u + e^- + \bar{\nu}_e$, $u + e^- \rightarrow d + \nu_e$ where $\mu_\Delta = \mu_d - \mu_u - \mu_e$ we find for the NFL dispersion relation eq. (1)

$$\Gamma_{dU}^{(\leftrightarrow)} \approx \frac{17G_F^2 \cos^2 \theta_c \alpha_s}{15\pi^2} \mu_q^2 \mu_e T^4 \left(1 + \sigma \log\left(\frac{\Lambda}{T}\right)\right)^2 \mu_\Delta \quad (4)$$

As observed previously in the study of the neutrino emissivity [7], this expression exhibits a similar but weaker NFL enhancement since only two quarks are present in the weak process. The same holds for the Cabbibo suppressed s -Urca channel. However, these Urca rates are strongly suppressed in T/μ compared to the non-leptonic rate eq. (3).

The bulk viscosity is a measure for the local dissipation caused by a (harmonic) oscillation of the conserved

	μ_Δ	B	C
quark non-lept.	$\mu_d - \mu_s$	$\frac{2\pi^2}{3\mu_q^2}$	$-\frac{m_s^2}{3\mu_q}$
quark d-Urca	$\mu_d - \mu_u - \mu_e$	$\frac{2\pi^2}{3\mu_q^2}$	$-\frac{m_d^2 - m_u^2}{3\mu_q}$
hadronic Urca	$\mu_n - \mu_p - \mu_e$	$\frac{8S}{n} + \frac{\pi^2}{(4(1-2x)S)^2}$	$4(1-2x)\left(n\frac{\partial S}{\partial n} - \frac{S}{3}\right)$

Table I: Strong interaction parameters, defined in eq. (5), describing the response of the particular form of dense matter. For comparison we also show the values for interacting hadronic matter in terms of the proton fraction x and the symmetry energy S . The corresponding weak rate for hadronic Urca processes is given in [12].

baryon density $n(\vec{r}, t) = \bar{n}(\vec{r}) + \Delta n(\vec{r}) \sin(\omega t)$ and can be defined in terms of the energy dissipation by

$$\zeta = \frac{2}{\omega^2} \left\langle \frac{d\epsilon}{dt} \right\rangle_{\text{diss}} \frac{\bar{n}^2}{(\Delta n)^2}.$$

Its detailed derivation is for instance discussed in [16]. Generally the non-leptonic and the Urca channels mix and require a coupled-channel analysis [22, 23], but due to the smallness of the Urca rate this effect is negligible and we evaluate them here individually. In the subthermal limit, denoted by the superscript “<”, the bulk viscosity has the general resonant form [16]

$$\zeta^< = \frac{C^2 \gamma}{\omega^2 + (B\gamma)^2} = \zeta_{max}^< \frac{2\omega B\gamma}{\omega^2 + (B\gamma)^2}$$

where the resonant maximum $\zeta_{max}^< = C^2/(2\omega B)$ is independent of the weak rate $\gamma \equiv \Gamma^{(\leftrightarrow)}/\mu_\Delta$, but the latter determines at which temperature this maximum is reached. Here the strong susceptibilities B and C determine the response of the medium and are given by

$$C \equiv \bar{n} \left. \frac{\partial \mu_\Delta}{\partial n} \right|_x, \quad B \equiv \frac{1}{\bar{n}} \left. \frac{\partial \mu_\Delta}{\partial x} \right|_n \quad (5)$$

where $x \equiv n_\Delta/\bar{n}$ is the fraction of the particle density corresponding to μ_Δ that is driven out of chemical equilibrium. The susceptibilities are given for the different processes in table I and in particular they do not involve logarithmic corrections. This can be seen from the pressure at finite temperature and density which has been computed in weak coupling beyond leading order in [24]. This result shows that all logarithmic temperature corrections are further suppressed in T/μ . This is generally expected since in the pressure and the susceptibilities the entire Fermi sea contributes and the contribution from the thin shell of width $O(T)$ around the Fermi surface is subleading.

Let us first consider the expected size of the NFL corrections. From our knowledge of the running of the strong coupling [25] at scales $O(\mu)$, quark matter at neutron star densities is, analogous to the strongly correlated

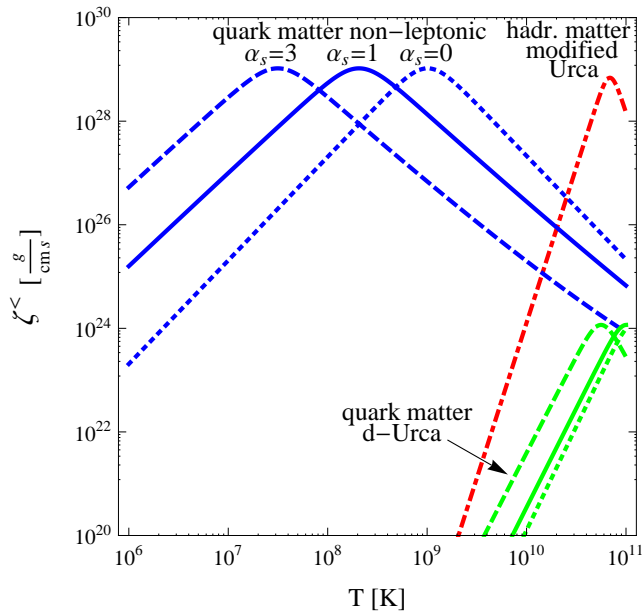


Figure 1: The influence of NFL effects due to long-ranged magnetic gauge interactions on the bulk viscosity of dense quark matter. All curves are given for a baryon density $n = 3n_0$ and a frequency $\nu = 1$ kHz. Shown is the dominant contribution due to non-leptonic processes for three values of the strong coupling constant ($\alpha_s = 0$ dotted, $\alpha_s = 1$ solid and $\alpha_s = 3$ dashed), as well as the subleading contribution from d -quark Urca reactions. For comparison the bulk viscosity due to modified Urca interactions in dense APR neutron star matter, studied in [16], is shown as well.

fluid observed at RHIC, expected to be strongly coupled $\alpha_s = O(1)$. Using the perturbative expressions eq. (2) as a guide, the relevant range of the parameters eq. (2) in compact stars should be roughly $0.05 \lesssim \sigma \lesssim 0.3$ and $100 \lesssim \Lambda/T \lesssim 10^6$ so that the enhancement factor λ^4 can take values in the range $2 \lesssim \lambda^4 \lesssim 1000$. Whereas the logarithmic form eq. (1) is generic, the perturbative relations between the parameters σ and Λ and the coupling do not need to hold, but due to our ignorance of the actual dependence in the strong-coupling regime we extrapolate them below.

Our results for the bulk viscosity are shown in fig. 1 for different values of the strong coupling, using an intermediate value for the strange quark mass $m_s = 150$ MeV. As can be seen the increase of the non-leptonic rate due to long-range gluonic interactions shifts the resonant maximum of the bulk viscosity to significantly lower temperatures and thereby strongly increases the damping of cold quark matter. As observed before in [22, 23] the Urca contribution is only relevant at small frequencies and temperatures $T > 10^9$ K. Due to the moderate change of the Urca rate this conclusion is unaltered by NFL effects. Compared to the corresponding bulk viscosity in hadronic matter the damping of interacting quark matter

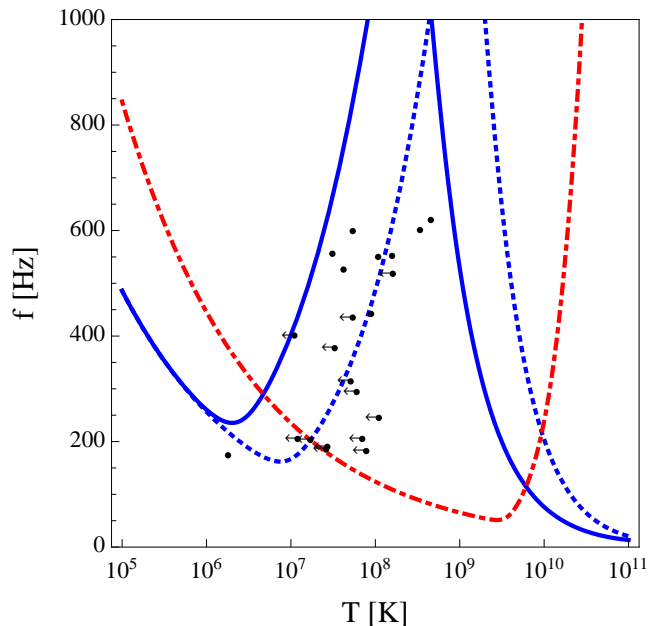


Figure 2: Impact of the NFL corrections to the bulk viscosity on the r-mode instability region for a $1.4 M_\odot$ strange quark star. The latter is shown for two values of the strong coupling constant ($\alpha_s = 0$ dotted and $\alpha_s = 1$ solid). For comparison the instability region of a $1.4 M_\odot$ neutron star is shown by the dot-dashed curve. For details on the star models see [26]. The theoretical curves are compared to LMXB pulsar data compiled in [27], where either temperature estimates are known (points) or only upper bounds were possible (points with arrows).

is more than ten orders of magnitude larger at temperatures $T < 10^8$ K present in cores of old compact stars.

As an example for the importance of the considered strong interaction corrections to the bulk viscosity of quark matter we study the instability of r-modes [28, 29] of strange quark stars [30]. R-modes are global oscillation modes of stars rotating at a finite angular velocity Ω , that are unstable to the emission of gravitational waves within a characteristic instability region in the T - Ω -plane. At the boundary of the instability region the gravitational instability is precisely balanced by viscous damping whereas for frequencies above the r-mode grows exponentially, for more details see [26]. If r-modes are unstable they strongly radiate gravitational waves and would quickly spin down a star [31]. Therefore, if the r-mode growth is stopped at values predicted by known saturation mechanisms [18, 32], observed low mass x-ray binaries (LMXBs), that are slowly spun up over long time periods by mass transfer from a companion, should not lie in the r-mode instability region. Yet, as can be seen in fig. 2 most observed LMXBs lie clearly in the instability region of a $1.4 M_\odot$ neutron star with standard damping mechanisms, i.e. shear viscosity due to leptons [11] and bulk viscosity due to modified Urca reactions

[12]. The situation is even worse for heavier stars and this conclusion cannot be changed by the involved uncertainties due to a remarkable insensitivity to the underlying parameters [26]. The enhanced dissipation of phases with ungapped quarks strongly increases the damping at low temperatures and leads to a stability window where r-modes are stable up to large frequencies. Nevertheless, neglecting interactions in the bulk viscosity many observed LMXBs are still within the instability region of a $1.4 M_{\odot}$ strange star, as shown by the dotted curve. However, taking into account long-range gauge interactions in the strong coupling case $\alpha_s = 1$ all observed pulsars are consistent with lying outside of the instability region. Therefore, the enhanced damping of ungapped quark matter can explain the large spin rates of LMXBs.

The results found here for strange stars immediately generalize to hybrid stars, where a quark matter core is enclosed in a hadronic outer part. Since the component with the stronger viscosity dominates the damping the instability region of a hybrid star is roughly given in each case by the upper segments of the neutron star and strange star curves [26]. Further our results apply to certain color superconducting phases that feature ungapped quark species and unscreened gluonic modes that can induce a NFL enhancement, see [7]. However, in hyperonic stars [33] where similar non-leptonic processes exist, there is no NFL enhancement since long range color interactions are absent.

It is striking that there are no data points for pulsars spinning with frequencies of several hundred Hertz that at the same time have core temperatures $\lesssim 10^7$ K. A stability window, as found here for ungapped quark matter, provides a robust explanation for this observation since the strong dissipative heating [34] due to r-modes in the instability region prevents stars from entering it. Clearly, our results involve considerable uncertainties on the detailed strong coupling values of the various low energy parameters entering the bulk viscosity. Nevertheless, within the uncertainties interacting quark matter can provide a consistent explanation for the astrophysical data, whereas hadronic matter with standard damping mechanisms can not [26]. Potential enhanced damping mechanisms in hadronic stars related to the complicated neutron star crust have been proposed, like boundary layer rubbing or mutual friction in hadron superfluids, see e.g. [27]. It requires detailed further study to decide if these mechanisms can likewise explain the astrophysical data, or if the data might eventually provide a robust signature for deconfined quark matter. In this endeavor the connection of the dynamic star evolution to pulsar timing data [34] should provide important information to distinguish between these different scenarios.

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